CLASS: PHY_____

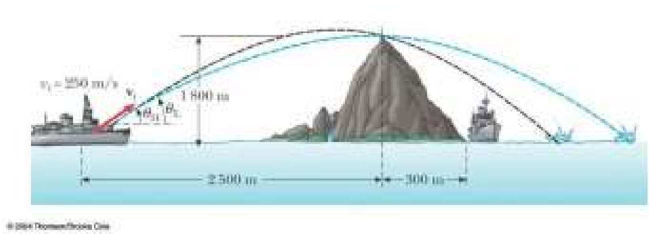
STUDENT #: ________________

NAME: ______________________

Assignment 3: Kinematics and Forces

Assigned: Sept 23 14:30 Due: September30 19:00

1 An enemy ship is on the east side of a mountain island, as shown. The enemy ship has maneuvered to within 2 500 m of the 1 800-m-high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the western shoreline is horizontally 300 m from the peak, what are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?



Find the highest firing angle θ_H for which the projectile will clear the mountain peak;
this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.
We may use the equation for trajectory obtained during our lecture:

$$y = (\tan\theta)x - \frac{g}{2v_o^2\cos^2\theta}x^2 \text{ but } \frac{1}{\cos^2\theta} = \tan^2\theta + 1 \text{ so}$$

$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$. Substitute values, use the quadratic formula and find $\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$.

Range at θ_H $R = \frac{v_o^2\sin 2\theta}{g} = 3070$ from enemy ship

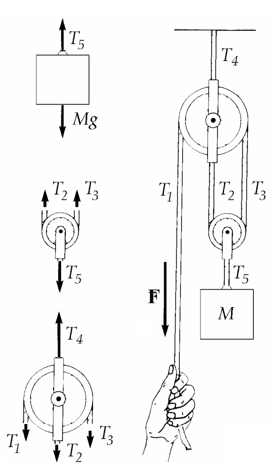
$3.07\times10^3 - 2\,500 - 300 = 270$ m from shore.

Range $R = \frac{v_o^2\sin 2\theta}{g} = 6280$ from enemy ship

Range at θ_L $6.28\times10^3 - 2\,500 - 300 = 3.48\times10^3$ from shore.

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48\times10^3 \text{ m}}$ from the shore.

2 An object of mass *M* is held in place by an applied force **F** and a pulley system as shown in Figure P5.55. The pulleys are massless and frictionless. Find (a) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5 and (b) the magnitude of **F**. *Suggestion:* Draw a free-body diagram for each pulley.



First, we note that $F = T_1$. Next, we focus on the mass M and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.

Since the pulleys are not starting to rotate and are frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2T_2$, so $T_2 = \frac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \frac{Mg}{2}}$, and $\boxed{T_4 = \frac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(b) Since $F = T_1$, we have $\boxed{F = \frac{Mg}{2}}$.

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Principles of Physics
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Dr. A. Czajkowski

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Assignment 3: Forces
CONT

Assigned: *Sept 23 14:30* *Due: September 30 19:00*

3 A 1.00-kg glider on a horizontal air track is pulled by a string at an angle θ. The taut string runs over a pulley and is attached to a hanging object of mass 0.500 kg as in Fig. P5.65. (a) Show that the speed v_x of the glider and the speed v_y of the hanging object are related by $v_x = uv_y$, where $u = z(z^2-h_0^2)^{-1/2}$. (b) The glider is released from rest. Show that at that instant the acceleration a_x of the glider and the acceleration a_y of the hanging object are related by $a_x = ua_y$. (c) Find the tension in the string at the instant the glider is released for h_0 = 80.0 cm and θ = 30.0°.

(a) Let x represent the position of the glider along the air track. Then $x = \sqrt{z^2 - h_0^2}$, so $\frac{dx}{dt} = \frac{z}{\sqrt{z^2 - h_0^2}}\frac{dz}{dt}$ Now $\frac{dz}{dt}$ is the rate at which string passes over the pulley, so it is equal to v_y of the counterweight.

$$\frac{dz}{dt} = \frac{dy}{dt} = v_y \quad \text{and so} \quad v_x = \frac{z}{\sqrt{z^2 - h_0^2}} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$ at release from rest, $v_y = 0$ and $a_x = ua_y$.

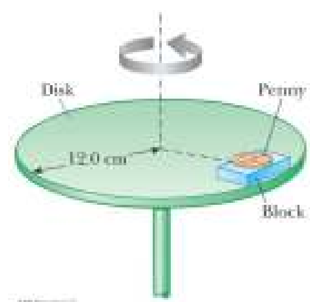
(c) $\sin 30.0° = \frac{80.0 \text{ cm}}{z}$, $z = 1.60$ m ,. $u = \frac{1.6}{\sqrt{1.6^2 - 0.8^2}} = 1.15$

For the counterweight $\sum F_y = ma_y$: $T - 0.5\text{ kg } 9.8\text{ m/s}^2 = -0.5\text{ kg}a_y$

$$a_y = -2T + 9.8$$

$$\sum F_x = ma_x : T\cos 30 = (1kg)a_x = (1.15kg)a_y$$
$$T\cos 30 = (1.15kg)(-2T + 9.8)$$
$$T(\cos 30 + 2.3) = 1.15(9.8) \rightarrow T = 3.56N$$

5 A penny of mass 3.10 g rests on a small 20.0-g block supported by a spinning disk. The coefficients of friction between block and disk are 0.750 (static) and 0.640 (kinetic) while those for the penny and block are 0.520 (static) and 0.450 (kinetic). What is the maximum rate of rotation in revolutions per minute that the disk can have, without the block or penny sliding on the disk?



For penny

$$\sum F_r = m\frac{v_{max}^2}{r}$$
$$\mu n = m\frac{v_{max}^2}{r}$$
$$\mu mg = m\frac{v_{max}^2}{r}$$
$$v_{max} = \sqrt{\mu gr}$$
$$v_{max} = \sqrt{0.520(9.8)(0.12)}$$

For block

$$\sum F_r = m\frac{v_{max}^2}{r}$$
$$0.750(0.020 + 0.00310)g + 0.520(0.0031)g = 0.020\frac{v_{max}^2}{0.12}$$
$$\sqrt{\frac{[0.750(0.020 + 0.00310) + 0.520(0.0031)](9.8)(0.12)}{0.020}} = v_{max}$$

Since the lower value is limiting value we get and since we get 0.78 m/s for the penny and 1.06 m/s for the block.
The maximum rate is 62.23rev/min (determined by the velocity at which the penny slips)